GEOMAGNETIC LATITUDES AND LONGITUDES OF SOURCE LOCATIONS OF PLANETARY RADIO EMISSIONS: THEORETICAL APPROACH AND SPACECRAFT OBSERVATIONS

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Abstract

We introduce a mathematical concept to determine source locations of planetary radio emissions based on spacecraft observations. We assume a constant frequency of an electromagnetic wave which is due to the electron cyclotron frequency. Furthermore, the magnetic field is approximated by a dipole and the emission of the wave is nearly perpendicular to the field. Therefore, one can define a constant magnetic field strength via the observed frequency which is used throughout our calculations. We derive magnetic latitudes and longitudes as functions of the distance, the angle between the dipole axis and the contour of constant magnetic field strength, and the cone angle of the emitted electromagnetic wave. We further apply our theoretical approach to single point measurements of the INTERBALL/AKR–X experiment observed at a frequency of 1463 kHz which corresponds to the so–called Sub Auroral Non–thermal Radio Emissions.

1 Introduction

Non–thermal planetary radio emission is a phenomenon of planetary magnetospheres and is understood as wave–particle interaction, where kinetic energy of electrons is transformed into electromagnetic energy. This interaction leads to the generation of electromagnetic waves which propagate under certain circumstances into interplanetary space. Using the kinetic theory of plasmas, Wu and Lee [1979] derived the mathematical concept of the

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so–called Cyclotron Maser Instability (CMI) which provides a concept of wave growth in the environment of a magnetoplasma. Their theory is based on the nearly perpendicular propagation of the electromagnetic wave (right–handed extraordinary wave mode) with respect to the magnetic field. Furthermore, they assumed that the refraction rate is approximately unity and that the frequency of the emitted electromagnetic wave is close to the local cyclotron frequency of electrons.

In this paper we concentrate on the so–called Sub Auroral Non–thermal Radio Emissions (SANE), first detected by Kuril'chik et al. [1988]. We note that no ground–based observations of this kind of radio emissions are possible, because in this specific frequency range of about 1.5 MHz, the Earth's ionosphere in between the source location and ground prohibits the propagation of electromagnetic waves downwards to the planet. Boudjada et al. [1998] analyzed SANE emissions at a frequency of 1463 kHz. In their report they examined the occurrence probability of SANE emissions but gave no detailed information about magnetic latitude and longitude of source locations.

For the performance of the mathematical concept of the problem, we assume that the observed frequency of 1463 kHz is due to the electron cyclotron frequency. This allows us to define a constant magnetic field strength, which is used throughout the calculations. We derive all possible magnetic latitudes for the SANE frequency as function of the distance to Earth. Next, we examine a second order equation to obtain the magnetic longitude as a function of two angles, the angle between the dipole axis and the contour of the constant magnetic field at 1463 kHz, and the cone angle of the emitted electromagnetic wave, following the work done by Vogl et al. [2001b]. As example we use four events of INTERBALL/AKR–X spacecraft data [Boudjada et al., 1998].

2 Mathematical model and basic equations

The cyclotron frequency of the electrons is given by the observed frequency of an electromagnetic wave f_c, the unit of charge ($e = 1.6 \times 10^{-19}$ C), and the mass of an electron ($m = 9.1 \times 10^{-31}$ kg),

$$\omega_c = 2\pi f_c = \frac{eB}{m}. \tag{1}$$

In zeroth order approximation, the Earth's magnetic field is described as an axis–symmetric dipole field [Kertz, 1969],

$$B = \frac{M_d}{r_1^3}\sqrt{1 + 3\cos^2\theta}. \tag{2}$$

Quantity M_d is the momentum of the magnetic dipole, given as $M_d = 0.3\Gamma R_E^3$, and R_E denotes the radius of the Earth. The angle θ defines the angular distance from the magnetic dipole axis to a point at an arbitrary magnetic field line, and quantity r_1 denotes the distance to the center of the Earth, introduced as

$$r_1 = r_0 \sin^2\theta, \tag{3}$$

where r_0 is the distance of the vertex of the field line to the center of the dipole.

3 Localization of the geomagnetic latitude

Assuming that the observed frequency of the electromagnetic wave does not change its value on the way to the observer, one uses equation (1) to obtain the relevant magnetic field strength, i.e.,

$$B_c = \frac{2\pi f_c m}{e}, \tag{4}$$

where f_c denotes the cyclotron frequency set equal to any observed frequency, in our case set equal to the frequency of SANE emissions, $f_c = 1463$ kHz. Substitution of equation (3) into equation (2) leads to an equation of sixth order in quantity r_1, i.e.,

$$r_1^6 + 3\frac{r_1}{r_0}\left(\frac{M_d}{B_c}\right)^2 - 4\left(\frac{M_d}{B_c}\right)^2 \equiv r_1^6 + 3\sin^2\theta\left(\frac{M_d}{B_c}\right)^2 - 4\left(\frac{M_d}{B_c}\right)^2 = 0, \tag{5}$$

which has to be solved for arbitrary θ ($0° \le \theta \le 90°$).

4 Localization of the geomagnetic longitude

The next step is to calculate the azimuth displacement of the magnetic field line, expressed by the angle φ. The geometric situation is sketched in Figure 1. Point P is the intersection of the magnetic field line and the isoline for a constant magnetic field, B_c, mathematically described by the position vector $\mathbf{r}_1$ and the two angles θ and φ. Point s/c denotes the position of the spacecraft, determined by position vector $\mathbf{r}_2$ and two angles, the polar distance α and the azimuth displacement of the satellite's position β.

The angle γ defines the vector product between the tangent vector, $\mathbf{T}$, and the vector of propagation of the electromagnetic wave, $\mathbf{S}$,

$$\cos\gamma = \frac{\mathbf{S}\cdot\mathbf{T}}{|\mathbf{S}||\mathbf{T}|}. \tag{6}$$

From Figure 1 it clearly turns out that the vector of propagation of the electromagnetic wave $\mathbf{S}$ is defined as $\mathbf{S} = \mathbf{r}_2 - \mathbf{r}_1$, i.e.,

$$\mathbf{S} = \begin{pmatrix} r_2\sin\alpha\cos\beta - r_1\sin\theta\cos\varphi \\ r_2\sin\alpha\sin\beta - r_1\sin\theta\sin\varphi \\ r_2\cos\alpha - r_1\cos\theta \end{pmatrix}. \tag{7}$$

We further define the tangent vector as

$$\mathbf{T} = \frac{\partial\mathbf{R}}{\partial\theta} = \begin{pmatrix} 3r_1\cos\varphi\cos\theta \\ 3r_1\sin\varphi\cos\theta \\ 2\frac{r_1}{\sin\theta} - 3r_1\sin\theta \end{pmatrix} = \begin{pmatrix} 3D\cos\varphi \\ 3D\sin\varphi \\ 2\frac{r_1^2}{\sqrt{r_1^2 - D^2}} - 3\sqrt{r_1^2 - D^2} \end{pmatrix}, \tag{8}$$

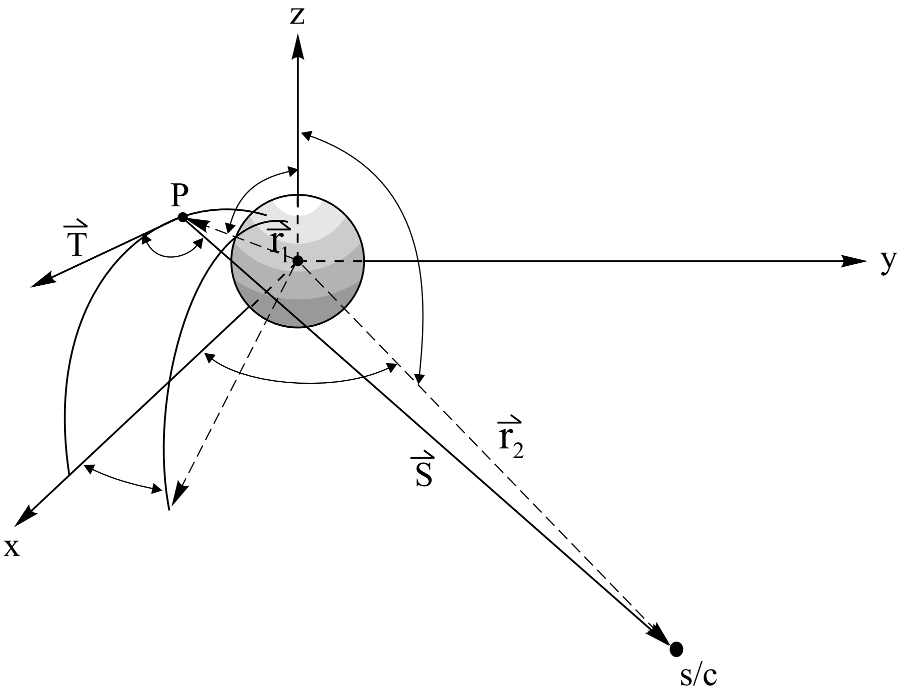

Figure 1: Sketch of the geometric situation of the problem.

where $D = r_1 \cos\theta$ and R denotes an arbitrary position vector in the spherical coordinate system. Substitution of equations (7) and (8) into equation (6) leads to a second order equation for the azimuth displacement, φ, [for more detail, see Vogl et al., 2001b]

$$\begin{aligned}\varphi_{1,2} \;=\;& \beta + \arccos\left\{\frac{-\cos\alpha\cos\theta}{\sin\alpha\sin\theta}\left(1-\frac{1}{3\cos^2\theta}\right)+\frac{2r_1}{3r_2\sin\alpha\sin\theta}\right.\\ & -\frac{r_1\cos^2\gamma}{3r_2\sin\alpha\sin\theta}\left(\frac{1}{3\cos^2\theta}+1\right)\pm\left[\frac{\cos\gamma\sqrt{1+3\cos^2\theta}}{9r_2\sin\alpha\sin\theta\cos^2\theta}\right.\\ & \left.\left.\times\sqrt{r_1^2\cos^2\gamma(1+3\cos^2\theta)-3\cos^2\theta(r_1^2-3r_2^2)-6r_1r_2\cos\alpha\cos\theta}\right]\right\}. \qquad (9)\end{aligned}$$

5 Data analysis

In this section, we apply our theoretical approach to INTERBALL spacecraft data, [Boudjada et al., 1998]. For analyzing the data we assume that the electromagnetic wave propagates perpendicular with respect to the magnetic field, $\gamma = 90°$. Thus, equation (9) reduces to

$$\varphi = \beta + \arccos\left[-\frac{\cos\theta\cos\alpha}{\sin\theta\sin\alpha}+\frac{\cos\alpha}{3\,\cos\theta\sin\theta\sin\alpha}+\frac{2r_1}{3\,r_2\sin\alpha\sin\theta}\right]. \qquad (10)$$

In particular we examine four events of INTERBALL spacecraft observations shown in

Table 1. Subscript g refers to geographic coordinates and subscript m to geomagnetic coordinates (β: longitudes, α: latitudes).

Event	Date	DOY	r_2 $[R_E]$	β_g	α_g	β_m	α_m
1	950814	226	2.3	162.3	39.0	-132.8	31.9
2	950921	264	5.0	164.0	61.5	-138.2	54.1
3	951010	283	19.4	151.0	62.0	-148.5	53.1
4	951229	363	5.7	142.0	64.2	-155.9	54.4

Table 1: Satellite positions where SANE emissions have been observed, from August, 1995 till December, 1995 (subscripts g and m refer to geographic and geomagnetic angles, respectively).

Figures 2 and 3 show possible source locations of DOY 226 and DOY 264. For the first event (DOY 226) the source might be located between 03:24 LT and 06:05 LT. Similar, Figure 4 and 5 show possible source locations for radio observations on DOY 283 and DOY 363.

6 Conclusions

We have introduced a simple mathematical concept to determine source locations of planetary radio emissions. This approach can be used for any arbitrary frequency and is based on geometric considerations. In obtaining the relevant equations we use the concept of constant magnetic field strength. This approach implies the fact that the observed frequency of an electromagnetic wave is set equal to the cyclotron frequency of electrons (CMI–theory). We further assume that the electromagnetic wave is emitted perpendicular to the magnetic field. We use experimental data observed by the AKR–X experiment on board of the INTERBALL spacecraft [Boudjada et al., 1998]. Table 2 summarizes the results for the azimuth (φ) and latitude (θ) ranges.

Event	φ [deg]	θ [deg]
DOY 226	-70.0 to -110.0	47.0 to 75.0
DOY 264	-77.0 to -42.0	45.1 to 52.6
DOY 283	-63.3 to -56.2	45.05 to 46.43
DOY 363	-87.5 to -86.0	45.4 to 49.0

Table 2: Possible source locations (azimuth (φ) and latitude (θ) ranges) for AKR–X experimental data.

Last, it has to be noted that our approach, which corresponds to the CMI–theory, is not applicable for the experimental data observed at 1463 kHz. As a matter of fact, this specific frequency leads to source locations very close to the surface of the Earth.

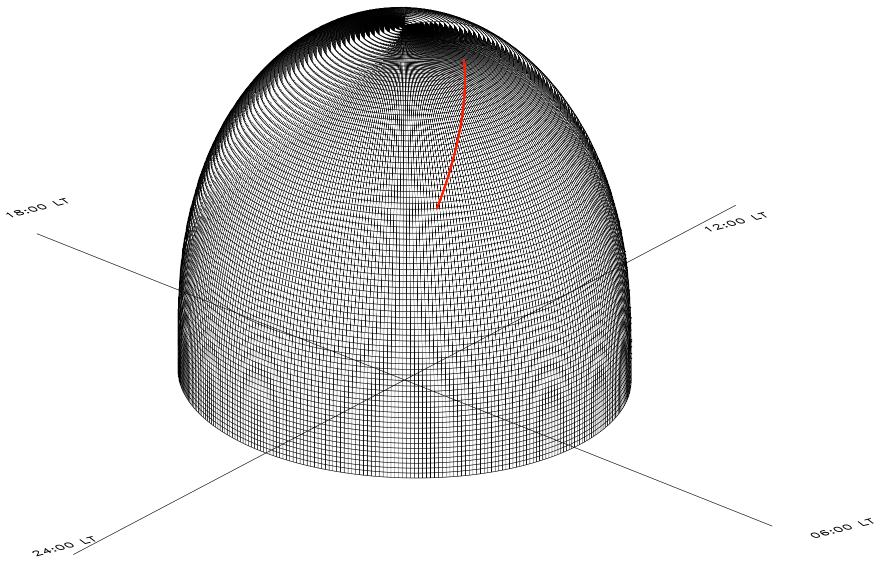

Figure 2: Possible solutions of the geographic latitude and longitude for radio observations at $f = 1463\,kHz$ on August 14, 1995, DOY=226.

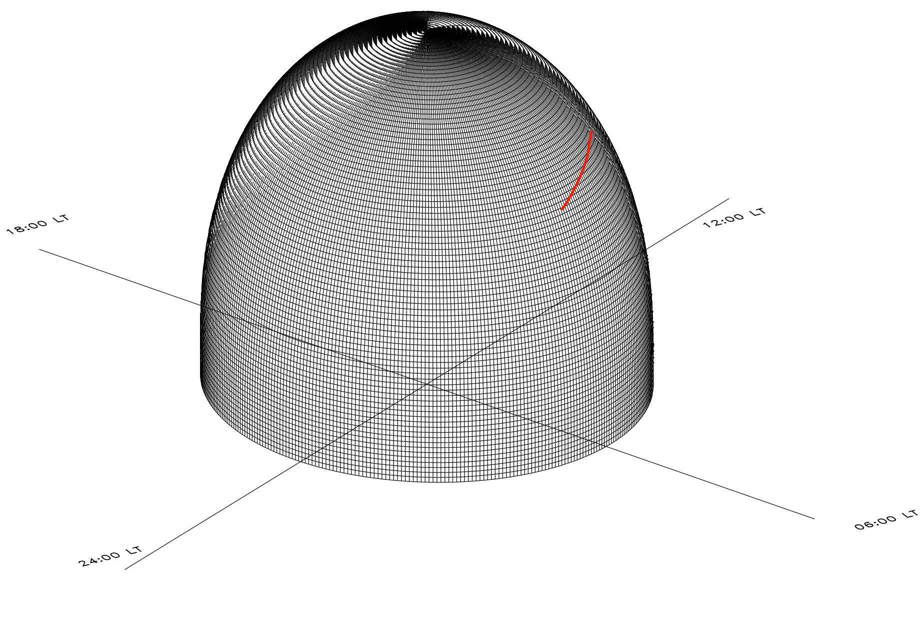

Figure 3: Possible solutions of the geographic latitude and longitude for radio observations at $f = 1463\,kHz$ on September 21, 1995, DOY=264.

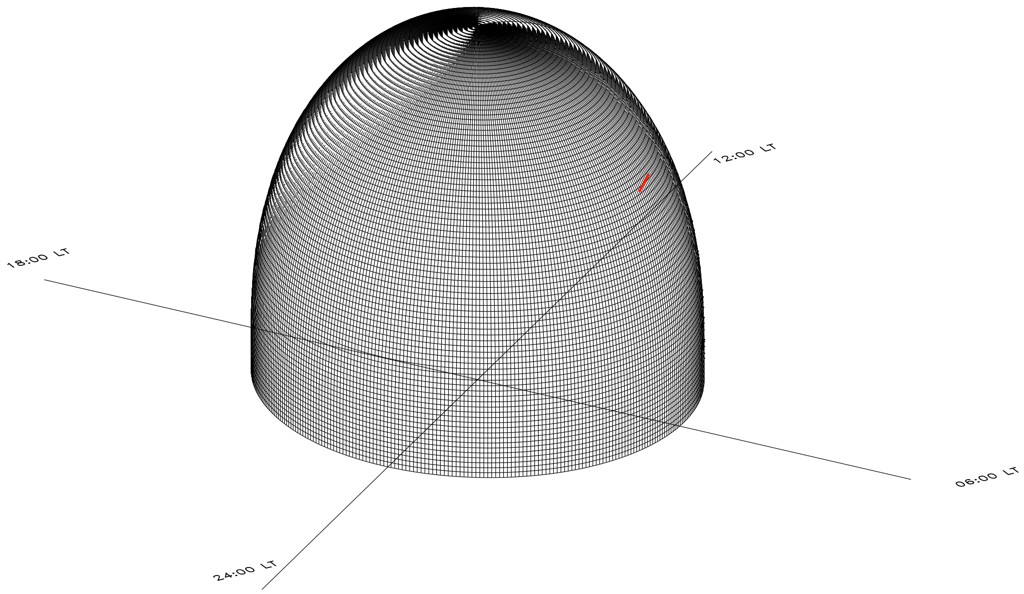

Figure 4: Possible solutions of the geographic latitude and longitude for radio observations at $f = 1463$ kHz on October 10, 1995, DOY=283.

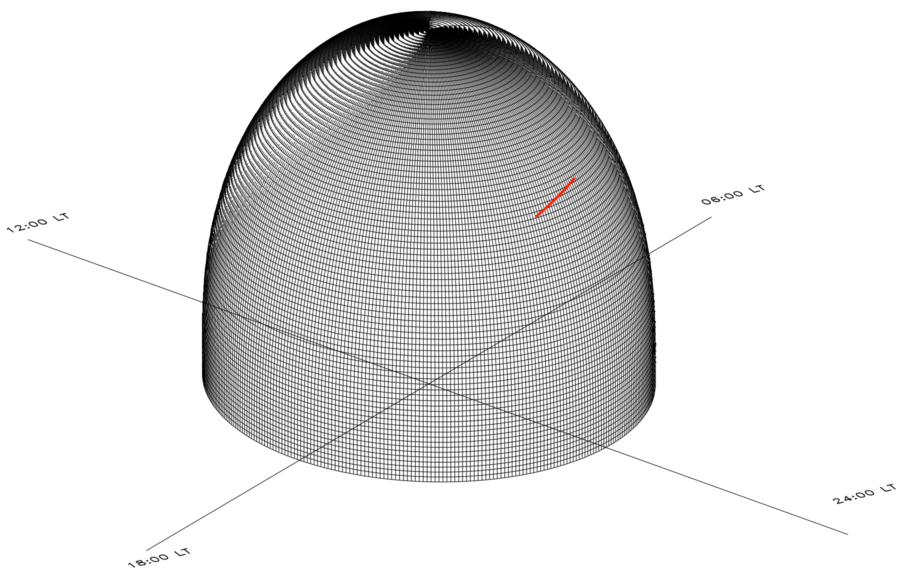

Figure 5: Possible solutions of the geographic latitude and longitude for radio observations at $f = 1463$ kHz on December 29, 1995 DOY=363.

Therefore, mode conversation must play an important role and the concept of the upper hybrid frequency may be relevant for this specific kind of radio emission.

In this mathematical concept the magnetic field was approximated by a dipole. A more realistic model can be obtained if the magnetic field is described by spherical harmonics. To obtain the possible source locations shown in Figures 2–4 it is assumed that the wave is propagating perpendicular to the magnetic field ($\gamma = 90°$). In dependence of magnetoplasma conditions the angle γ can be less than 90° which will also lead to a more realistic point of view. As noted above, this approach is not applicable to SANE observations. The generation of SANE is not in line with the CMI–theory, therefore other generation mechanisms where the emitted frequency, $f_{emission}$, depends on the electron density n_e, has to be taken into account. This means that the plasma frequency has to be included, i.e. $f_{emission}$ set equal to the upper hybrid frequency (f_{uh}). Another possibility to explain this type of emission might be the dependence of $f_{emission}$ on harmonics of the gyration frequency.

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